Computer Simulations of the TriAxys Directional Wave Buoy

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1. Introduction
This note describes some of the computer simulations that have been carried out at the NRC Canadian Hydraulics Centre (CHC) to evaluate the performance of the TriAxys directional wave buoy. These simulations have been performed for the latest version of the buoy that uses three Jewell LCF-200 accelerometers and three Murata ENV-05H angular rate sensors. The simulation results provide information on how accurately the buoy motions can be measured and how accurately the directional wave properties can be determined from the measured buoy motions. Simulations were performed for a variety of wave conditions with significant wave heights ranging from 1 to 16 metres and peak wave periods ranging from 5 to 20 seconds. Directional spectra with both narrow and broad spreading widths were simulated for a variety of mean wave directions.

2. Simulation Procedure
The simulations were performed using the CHC GEDAP software package. Standard GEDAP programs were used to synthesize random directional wave fields by the single summation method [1] for specified wave spectra \( S(f) \) and directional spreading functions \( D(f, \theta) \). The wave synthesis procedure produced time series records of wave elevation, x and y velocity components and x and y wave slopes.

A custom GEDAP program named AXYS_SIM5 was then used to compute the actual buoy motions that would be produced by the synthesized directional waves. This program uses linear transfer functions for each of the buoy motions. For example, the heave response is modeled as a second order mass-spring system with a natural frequency of 0.42 Hz and a damping ratio of 0.6. Since the buoy is small compared to the wave lengths, it just follows the orbital velocities of the waves over most of its operational frequency range. Thus, the heave motion is very similar to the wave elevation and the surge and sway velocities are the same as the north and west wave velocities resolved along the instantaneous x and y axes of the buoy as defined by the current yaw angle. Similarly, the roll and pitch angles of the buoy generally follow the local wave slope as resolved along the instantaneous x and y axes of the buoy. The yaw angle is modeled as a slow linear drift with a superimposed random oscillatory component. Various drift rates can be set to check the ability of the buoy to correctly measure directional waves when it is slowly turning about its z-axis.
Program AXYS~SIM5 produces 18 time series records defining displacement, velocity and acceleration for each of the actual buoy motions (roll, pitch, yaw, surge, sway and heave). Another program named MOTSYN4 is then used to compute the sensor signals that would result from these motions. For example, each accelerometer would measure the total acceleration due to the translational and rotational motions of the buoy plus an earth gravity component depending on the instantaneous roll and pitch angles. Similarly, each angular rate sensor produces a signal proportional to the instantaneous rotation vector of the buoy resolved along the sensitive axis of the sensor.

Program MOTSYN4 also adds Gaussian white noise to each of the sensor signals to simulate the noise levels of the actual sensors. The RMS noise levels were set to 0.05 degs for the angular rate sensors and 0.0005 g for the accelerometers. These are the expected noise levels for the sensors when sampled through a high quality 12-bit A/D converter with an accuracy of 11/2 LSB. These noise levels were set at twice the RMS noise levels of 0.025 degs and 0.00025 g that were obtained from the specifications and bench tests of the actual sensors using a 16-bit laboratory data acquisition system. The higher noise levels were used in the simulations to provide an allowance for additional sources of noise in the buoy, such as, the signal conditioning and power supply components.

These synthesized sensor signals were then used as input to programs MOTION4 and WAVAN4 that are used in the TriAxys Buoy for motion measurement and directional wave analysis. The output wave data from program WAVAN4 were also processed by program MEM4 that is used in the TriAxys buoy base station PC for directional wave analysis by the NRC Maximum Entropy Method.

The resulting motion data from MOTION4 was then compared to the actual synthesized buoy motions to determine how accurately the real motions could be computed from the synthesized sensor signals. Similarly, the wave analysis results from WAVAN4 and MEM4 were compared to the actual known characteristics of the directional waves that were used in the simulations.

3. Simulated Wave Conditions
Simulations were carried out for the eight test cases listed in Table 1. All waves were synthesized for a JONSWAP parent spectrum with 'y = 3.3. The D5 test cases had a narrow directional spreading of \( \cos^{28}(0) \) with \( S = 40 \) corresponding to a spreading width of \( \pm 12.7 \) deg. The D6 test cases had a broad directional spreading of \( \cos^{5}S(9) \) with \( S = 6 \) corresponding to a spreading width of \( \sim = 31.7 \) deg. The time series record length of the simulated sensor signals was set to 25 minutes for all of these test cases. The low frequency cut-off in program MOTION4 was also set to 0.030 Hz for all cases.
4. Simulation Results
The results of the eight simulated test cases are shown graphically in Figures 1 to 32. There are four figures for each test case.

Figures 1 and 2 show the buoy motions computed by program MOTION4 from the simulated sensor signals together with the corresponding actual buoy motions for test case D5R1. It can be seen that there is very close agreement for all six motions (roll, pitch, yaw, surge velocity, sway velocity and heave displacement). Although the total record length is 1500 seconds, these plots show a representative one-minute segment from 500 to 560 seconds.

Figure 3 shows the wave spectrum as would be computed by the TriAxys buoy plotted together with the actual wave elevation spectrum for the synthesized waves. The parent JONSWAP spectrum is also shown as a grey background. The difference between the actual wave spectrum and the parent spectrum is due to the normal statistical variability of the particular 25 minute synthesized wave field. Note that if the TriAxys buoy was working perfectly, then its measured spectrum should match the actual wave spectrum, not the parent JONSWAP spectrum. In other words, the actual spectrum is the spectrum of the particular 25-minute wave elevation record for which the buoy motions were simulated.

Figure 3 also lists the actual peak period $T_p$ and the actual significant wave height $H_{mo}$ together with the corresponding values computed by the TriAxys buoy software.

The measured mean wave direction and the spreading width $\beta$ are plotted as functions of frequency in Figure 4. These have been computed from the measured buoy motions by the KVH Method [2] and the NRC MEM Method [3]. The actual mean direction and spreading

<table>
<thead>
<tr>
<th>Test Case</th>
<th>$T_p$ (S)</th>
<th>$H_{mo}$ (M)</th>
<th>Mean Direction (degrees)</th>
<th>Spreading Width (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D5R1</td>
<td>5.0</td>
<td>1.0</td>
<td>322</td>
<td>12.7</td>
</tr>
<tr>
<td>D5R2</td>
<td>10.0</td>
<td>4.0</td>
<td>55</td>
<td>12.7</td>
</tr>
<tr>
<td>D5R3</td>
<td>15.0</td>
<td>9.0</td>
<td>132</td>
<td>12.7</td>
</tr>
<tr>
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<td>20.0</td>
<td>16.0</td>
<td>160</td>
<td>12.7</td>
</tr>
<tr>
<td>D6R1</td>
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<td>1.2</td>
<td>310</td>
<td>31.7</td>
</tr>
<tr>
<td>D6R2</td>
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<td>40</td>
<td>31.7</td>
</tr>
<tr>
<td>D6R3</td>
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<td>127</td>
<td>31.7</td>
</tr>
<tr>
<td>D6R4</td>
<td>18.0</td>
<td>10.0</td>
<td>163</td>
<td>31.7</td>
</tr>
</tbody>
</table>

Table 1: Simulated Wave Conditions
width for the synthesized wave field are also shown as grey lines in this figure together with
dashed grey lines indicating the target values of 322 deg and 12.7 deg, respectively. Note
that the actual mean direction and spreading width are not constant but have some
variability about the target values. This is the natural statistical variability due to the finite
record length and the finite number of random wave angles (8192) that were used in the
simulated wave field.

Figures 5 to 32 show the corresponding results for the other seven test cases.

It can be seen from Figures 4, 8, 12, 16, 20, 24, 28 and 32 that the KVH and MEM methods
give similar results for both the mean wave direction and the directional spreading width,
~o. The KVH and MEM results are computed over the frequency range from f1 to f2 where
these limits are set such that one percent of the total wave energy lies below f1 and one
percent of the total wave energy lies above f2. These frequency limits are used to ensure that
there is sufficient wave energy available to make meaningful measurements of the mean
wave direction and the spreading width. It can be seen that both the KVH and MEM results
match the actual mean direction and spreading width very well over this frequency range in
all of the simulated test cases.

In order to assess the effects of higher noise levels, runs DSR1 and D5R2 were repeated as
runs DSR5 and DSR6 with the RMS noise levels increased to 0.25 degs for the angular rate
sensors and 0.0025 g for the accelerometers. All other parameters were kept the same. The
results for runs D5RS and D5R6 are shown in Figures 33 to 40. A comparison of Figures 3
and 35 shows that the main effect of the higher noise levels on the wave energy spectrum
has been to cause a spurious low frequency peak at 0.035 Hz. It can be seen in Figure 36
that the higher noise levels also cause large errors in the mean direction and spreading width
at frequencies below 0.14 Hz where there is very little real wave energy and most of the
apparent wave energy is due to noise. At frequencies above 0.14 Hz, it can also be seen that
the noise has less effect on the MEM results than it does on the KVH results.

A comparison of Figures 7 and 39 also shows a spurious second peak at 0.035 Hz but it is
very small compared to the main wave spectrum. Consequently, as can be seen in Figure 40,
the higher noise has very little effect on the mean direction and the spreading width for this
case because of the f1 and f2 frequency limits that are used to compute these parameters.
Thus, the results for runs D5R5 and DSR6 show that sensor noise has relatively less effect
at higher significant heights as one would expect since the signal to noise ratio increases
with wave height for a given noise level. However, the results for run DSR5 clearly show
that the RMS noise levels must not be much larger than 0.05 degs and 0.0005 g to maintain
good performance down to 0.03 Hz when the significant wave height is only 1 m or so.
5. References


Figure 1: Roll, Pitch and Yaw Motions for Test Case D5R1
Figure 2: Surge, Sway and Heave Motions for Test Case D5R1
Figure 3: Comparison of wave spectra for Test Case D5R1
Figure 4: Directional wave analysis results for Test Case D5R1.
Figure 5: Roll, Pitch and Yaw Motions for Test Case D5R2
Test Case D5R2: $T_p = 10.0 \text{ s}$, $H_{mo} = 4.0 \text{ m}$, Mean Wave Direction = 55 deg

Figure 6: Surge, Sway and Heave Motions for Test Case D5R2
Figure 7: Comparison of wave spectra for Test Case D5R2
Figure 8: Directional wave analysis results for Test Case D5R2
Figure 9: Roll, Pitch and Yaw Motions for Test Case D5R3
Figure 10: Surge, Sway and Heave Motions for Test Case D5R3
Figure 11: Comparison of wave spectra for Test Case D5R3
Figure 12: Directional wave analysis results for Test Case D5R3
Figure 13: Roll, Pitch and Yaw Motions for Test Case D5R4
Figure 14: Surge, Sway and Heave Motions for Test Case D5R4
Figure 16: Directional wave analysis results for Test Case D5R4
Figure 17: Roll, Pitch and Yaw Motions for Test Case D6R1
Figure 18: Surge, Sway and Heave Motions for Test Case D6R1
Figure 19: Comparison of wave spectra for Test Case D6R1
Figure 20: Directional wave analysis results for Test Case D6R1
Figure 21: Roll, Pitch and Yaw Motions for Test Case D6R2
Figure 22: Surge, Sway and Heave Motions for Test Case D6R2
Figure 23: Comparison of wave spectra for Test Case D6R2
Figure 24: Directional wave analysis results for Test Case D6R2
Figure 25: Roll, Pitch and Yaw Motions for Test Case D6R3
Figure 26: Surge, Sway and Heave Motions for Test Case D6R3
Figure 27: Comparison of wave spectra for Test Case D6R3
Figure 28: Directional wave analysis results for Test Case D6R3
Figure 29: Roll, Pitch and Yaw Motions for Test Case D6R4
Figure 30: Surge, Sway and Heave Motions for Test Case D6R4
Figure 31: Comparison of wave spectra for Test Case D6R4
Figure 32: Directional wave analysis results for Test Case D6R4
Figure 33: Roll, Pitch and Yaw Motions for Test Case D5R5
Figure 34: Surge, Sway and Heave Motions for Test Case D5R5
Test Case D5R5: $T_p = 5.0$ s $H_{mo} = 1.0$ m Mean Wave Direction = 322 deg

Figure 35: Comparison of wave spectra for Test Case D5R5

- JONSWAP Parent Spectrum
- Actual Wave Spectrum
- TriAxys Buoy Spectrum

Actual $T_p = 4.96$ s
TriAxys Buoy $T_p = 4.94$ s
Actual $H_{mo} = 1.01$ m
TriAxys Buoy $H_{mo} = 1.03$ m
Figure 36: Directional wave analysis results for Test Case D5R5
Figure 37: Roll, Pitch and Yaw Motions for Test Case D5R6
Figure 38: Surge, Sway and Heave Motions for Test Case D5R6
Figure 39: Comparison of wave spectra for Test Case D5R6
Figure 40: Directional wave analysis results for Test Case D5R6